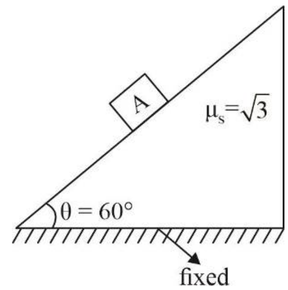
Solutions to JEE Advanced Home Practice Test -5 |JEE 2024 | Paper-2

Physics

ONE OR MORE THAN ONE CHOICE

1.(AD) For incline and block A, $\tan 60° = \mu_s$

So, at 60° the static friction is sufficient (with it maximum value) to stop the block 'A'.



For block 'B', the angle 60° is more than angle of repose (for friction between block A and B).

So, f_k will start acting between block A and B as block B will start slipping. $\Rightarrow (f_k) < (f_s)_{max}$

(between block 'A' and 'B')

So, $f_k < m_B g \sin\theta$

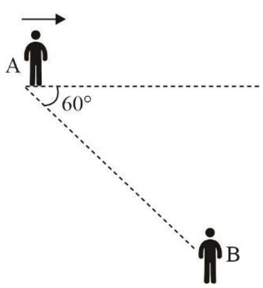
So, net force down the incline on block 'A' is

$m_A g \sin\theta + f_k = F_A$

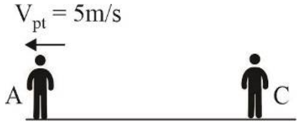
So, block A will be at rest only by static friction

So, $T = 0$

2.(AC) $U_{pg} = 105 - 100 = 100 m/s$



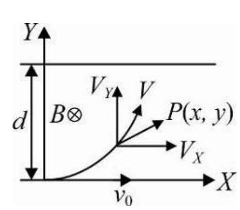
$$f = f_0 \frac{v}{(V - U_{pg}\cos 60°)} = \frac{f_0(350)}{(350 - 100 \times \frac{1}{2})} = f_0 \frac{350}{300} = \frac{7}{6} f_0$$



$$f = f_0 \frac{u}{(V + U_{pt})} = f_0 \frac{350}{(350+5)}; \qquad f = f_0 \frac{350}{355} = \frac{70}{71} f_0$$

3.(A) Let at time *t*, particle be at point $P(x, y)$ and its velocity be

$\vec{v} = (v_x \hat{i} + v_y \hat{j}) \qquad \Rightarrow \qquad v_0^2 = v_x^2 + v_y^2$



(work done by magnetic field is always zero, so there is no change in magnitude of velocity).

Then, magnetic force on the particle at point P is $\vec{F} = q(v_x\hat{i} + v_y\hat{j}) \cdot B_0\left(1+\frac{y}{d}\right)(-\hat{k})$

$$\Rightarrow \quad -qB_0\left[1+\frac{y}{d}\right]dy = mdv_x$$

Now, when the particle will be coming out of the at that point $y = d$.

Let the velocity in X-direction be v_x, then integrating we get

$$\int_{v_0}^{v_x} dv_x = -\frac{qB_0}{m}\int_0^d\left[1+\frac{y}{d}\right]dy = -\frac{qB_0}{m}\left[d+\frac{d^2}{2d}\right] = -\frac{3qB_0d}{2m}$$

So, $\quad v_x = v_0 - \frac{3qB_0d}{2m} \qquad \therefore \qquad k = 3$

4.(AD) $i_D =$ displacement current $= \varepsilon_0 A\frac{d\overline{E}}{dt}$

$$E_{ind} = L\frac{di}{dt} \Rightarrow di = \frac{E_{ind}\times dt}{L}; \qquad \tau = \overline{m}\times\overline{B} \Rightarrow i = \frac{\tau}{B\times A\times\sin\theta}$$

5.(AB) $Q = (M_x - M_y - M_e)c^2$

$KE_{\max}\beta^- = Q -$ excitation energy of Y^*

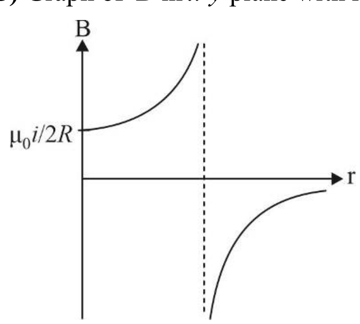
$= Q - E_y$

Smallest wavelength photon emitted by Y^*

$$\lambda = \frac{hc}{E_y}$$

Kinetic energy of atomic e^- emitted through internal conversion $= E_y - B$, where B is the binding energy of the e^-.

6.(ABC) Graph of $\overline{B}$ in x-y plane with radial distance r



Due to symmetry of loop, $\overline{B}$ in x-y plane only depends upon radial distance

Outside the loop B varies from ∞ to O, so there will be points with $B = \frac{\mu_0 i}{2R}$

In the graph its clear that B increases as we move from centre to circumference of loop

No point in side the loop has B = 0

COMPREHENSION WITH NUMERICAL TYPE

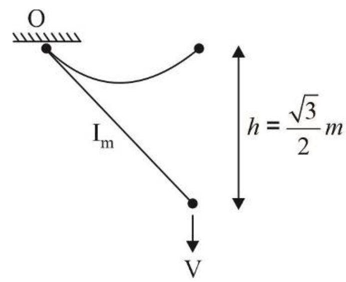
7.(1) Heat emitted by the surface of sphere per unit time

$$P_{r'} = \sigma T^4 (\pi R^2) E \,; \qquad \frac{P_{A_1}}{P_{lead}} = \frac{1}{1}$$

8.(39) $$\frac{P_1}{P_2} = \frac{m_1 S_1 \frac{d\theta_1}{dt}}{m_2 S_2 \frac{d\theta_2}{dt}} = \frac{V_1 d_1 S_1 \frac{d\theta_1}{dt}}{V_2 d_2 S_2 \frac{d\theta_2}{dt}} = 1$$

$$\frac{\frac{d\theta_1}{dt}}{\frac{d\theta_2}{dt}} = \frac{S_2}{S_1}\frac{d_2}{d_1} = \frac{130 \times 2.7}{900 \times 100} = \frac{39}{1000}$$

9.(4.15-4.16) & 10.(4.32-4.33)

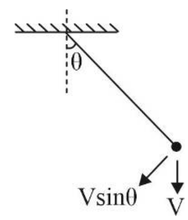


$$V = \sqrt{2gh} = \sqrt{2 \times 10 \times \left(\frac{\sqrt{3}}{2}\right)}$$

$$= \sqrt{17.3} \simeq 4.159\, m/s$$

$$L_0 = mV \times r_1 = 2 \times 4.159 \times 0.5 = 4.16\, kgm^2/s$$

When string becomes taut, $\theta = 30°$

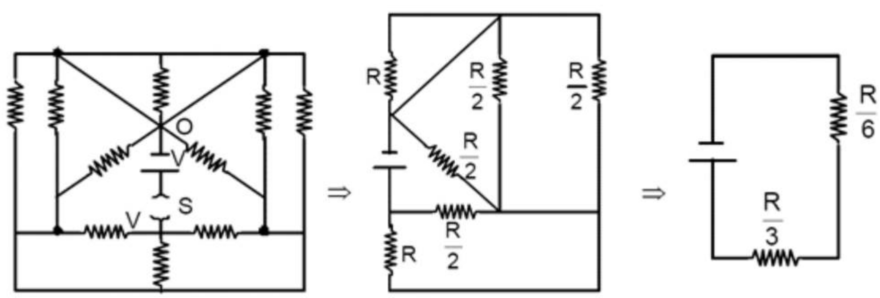


$$k = \frac{1}{2} m (V \sin\theta)^2 = \frac{1}{2} \times 2 \times \sqrt{3} \times 10 \times \frac{1}{4} = \frac{17.3}{4} = 4.32 \text{ Joules}$$

11.(2), 12.(1)

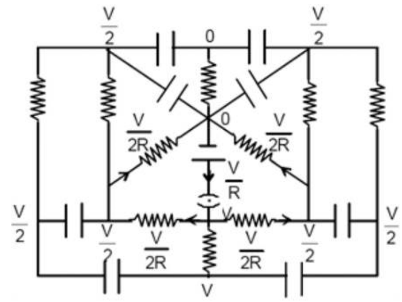
At $t = 0$ all the capacitor are short circuited. So current at $t = 0$ can be calculated as below.

$$i = \frac{V}{R/2} = \frac{2V}{R}$$



Now at $t = \infty$. Current through each capacitor reduce to zero.

So at $t = \infty$ circuit is:



COMPREHENSION WITH SINGLE TYPE

13.(B) $\phi = Li = \frac{\mu_0 m}{2\pi r^3} \times a^2 \quad \Rightarrow \quad i = \frac{\mu_0 m a^2}{2\pi r^3 L}$

$\Rightarrow i \propto \frac{m}{r^3}; \quad M = a^2 i = \frac{\mu_0 m a^4}{2\pi r^3 L}$

14.(D) $F = \frac{km^2 a^4 \mu_0}{2\pi r^7 L}$

15.(B) Work done in adiabatic compression is used to work against elastic force and against atmospheric force.

16.(C) Final pressure, $P_f = \left(P_0 + \frac{Kh/16}{A}\right)$

$$\frac{P_f V_f}{T_f} = \frac{P_0 V_0}{T_0}; \quad \frac{\left(P_0 + \frac{kh}{16A}\right) A \times \left(\frac{h}{2} + \frac{h}{16}\right)}{T_f} = \frac{P_0 A h}{T_0}$$

$$\frac{\left(10^5 + \frac{3700h}{16 \times 27 \times 10^{-4}}\right)\frac{9}{16}}{\frac{4}{3} \times 273} = \frac{10^5}{273}; \quad (1 + 0.856h) = \frac{16}{9} \times \frac{4}{3}$$

Solving, $h = 1.6m$

INTEGER TYPE

17.(2) It is the case of non-uniform circular motion

$$F_{tangential} = m\frac{dv}{dt} = m\frac{d(\omega l)}{dt} = mL\frac{dv}{dt} = m\alpha L$$

$= m\alpha L \quad \ldots(i)$

$F_{radial} = m\omega^2 L \quad \ldots(ii)$

Net force, $F_{net} = \sqrt{(m\alpha L)^2 + (m\omega^2 L)^2} \quad ; \quad f \le \mu N \quad \Rightarrow \sqrt{(m\alpha L)^2 + (m\omega^2 L)^2} \le \mu mg$

$$\alpha^2 L^2 + \omega^4 L^2 \le \mu^2 g^2 \Rightarrow \omega^4 \le \frac{\mu^2 g^2}{L^2} - \alpha^2 \Rightarrow \omega \le \left[\left(\frac{\mu g}{L}\right)^2 - \alpha^2\right]^{1/4}$$

18.(3) $F = \frac{-k}{r^2} \quad \therefore \quad$ Potential energy $= \frac{-k}{r}$

Now applying conservation of energy at a & b

$\frac{1}{2}MV_1^2 - \frac{K}{a} = \frac{1}{2}MV_2^2 - \frac{K}{b} \quad \ldots(i)$

$V_1 = \sqrt{\frac{K}{2Ma}}$ (given) $\quad \ldots(ii)$

Applying conservation of angular momentum

$MV_1 a = MV_2 b$

$$V_2 = \frac{a}{b}V_1 = \frac{a}{b}\sqrt{\frac{K}{2Ma}} \qquad \ldots\text{(iii)}$$

Now putting (ii) and (iii) in (i) we get $\frac{a}{b} = 3$

19.(22) The target area is $S_1 = \pi(10^{-9})^2 = \pi \times 10^{-18} m^2$.

The area of a 5 m sphere centered on the light source is, $S_2 = 4\pi(5)^2 = 100\pi m^2$. Thus, if the light source radiates uniformly in all directions, the rate P at which energy falls on the target is given by

$$P = (10^{-3} watt)\left(\frac{S_1}{S_2}\right) = (10^{-3})\left(\frac{\pi \times 10^{-18}}{100 \times \pi}\right) = 10^{-23} J/s$$

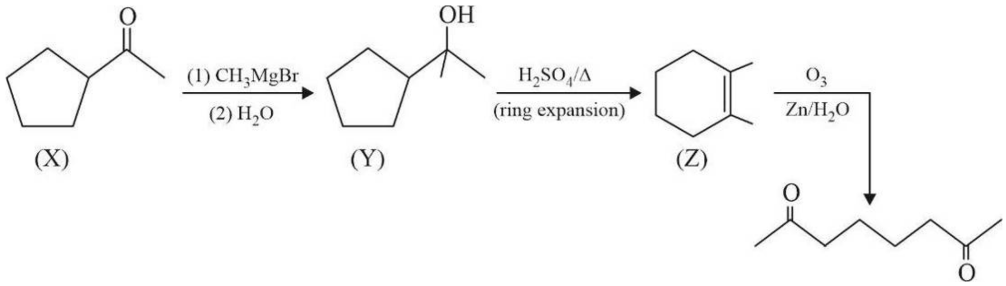
Assuming that all power is absorbed, the required time is

$$r = \left(\frac{5eV}{10^{-23} J/s}\right)\left(\frac{1.6 \times 10^{-19} J}{1eV}\right) = 22\,hrs$$

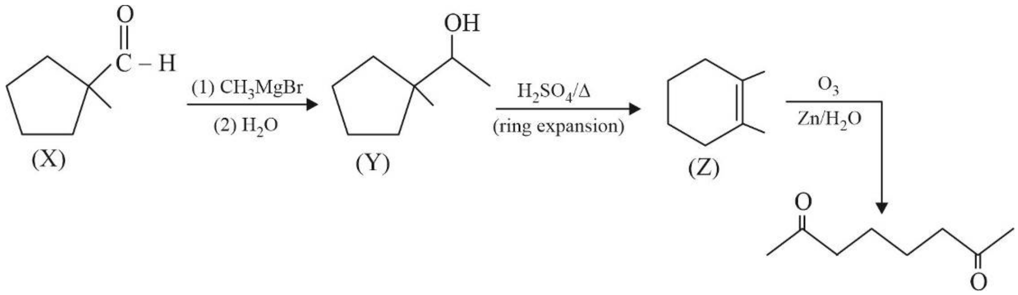
Chemistry

ONE OR MORE THAN ONE CHOICE

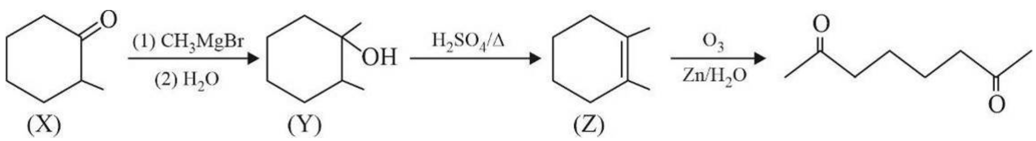
1.(ABC) **(A)**



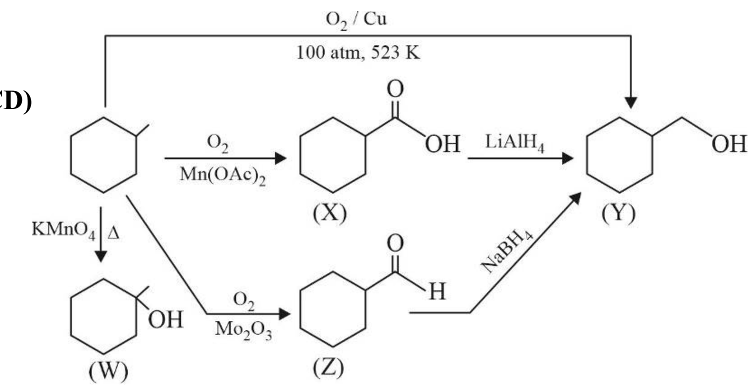
(B)



(C)



2.(ACD)



3.(BD) Using Exp. 1 & 2, we get a = 0.5

Using Exp. 1 & 3, we get b = 1

Using Exp. 2 &4, we get c = 1.5

$\therefore \quad r = k[X]^{1/2}[Y][Z]^{3/2}$

At $[X] = 0.25M, [Y] = 0.02M \ \& \ [Z] = 0.09M$

$r = 10^{-2}[0.25]^{1/2}[0.02][0.09]^{3/2} = 27 \times 10^{-7}\,Ms^{-1}$

$R = 27 \Rightarrow \sqrt[3]{R} = 3$

4.(BCD) S.R.P ↑ O.A ↑ S.R.P = –S.O.P

$E^{\circ}_{Ce^{+4}|Ce^{+3}} > E^{\circ}_{MnO_4^-|Mn^{+2}} > E^{\circ}_{Cr_2O_7^{-2}|Cr^{+3}} > E^{\circ}_{Sn^{+4}|Sn^{+2}}$

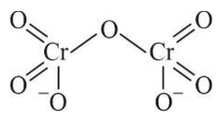
1.61 1.51 1.33 0.15

5.(ACD) $[Pt(ox)_3]^{2-}$ is optically active due to 3 bidentate ligands and absence of plane of symmetry. $[PdCl_4]^{2-}$ and $[Pt(ox)_3]^{2-}$ do not show G.I. as all ligands are same.

Hybridization of Ni^{2+} is dsp^2 in $[Ni(CN)_4]^{2-}$ and that of Hg^{2+} is sp^3 in $[Hg(SCN)_4]^{2-}$ Both CN^- and SCN^- ligands are ambidentate. Hence both $[Ni(CN)_4]^{2-}$ and $[Hg(SCN)_4]^{2-}$ show linkage isomerism. $K_2[Ni(CN)_4]$ is diamagnetic but $Co[Hg(SCN)_4]$ is not as Co^{2+} is d^7 – paramagnetic.

6.(ABCD) (A) HCl gets itself oxidised to chlorine in permanganate titrations

(B) $3MnO_4^{-2} + 4H^+ \longrightarrow 2MnO_4^- + MnO_2 + 2H_2O$

(C) 

(D) The colour of the complex ion is attributed to the presence of unpaired electron which is/are able to under go d-d transition in visible light. Ti^{2+} has two unpaired electrons. So, d-d transition is possible, Ti^{4+} has no electrons in the d-orbitals.

COMPREHENSION WITH NUMERICAL TYPE

7.(0.19) For $25\times10^{-4}(M)$ NaCl solution

$$\lambda_m = \lambda_m^\infty - b\sqrt{C}$$

$$107 = \lambda_m^\infty - b\sqrt{4\times10^{-4}} \qquad \ldots(i)$$

$$97 = \lambda_m^\infty - b\sqrt{9\times10^{-4}} \qquad \ldots(ii)$$

Solving above equations

$\lambda_m^\infty = 127$ and $b = 1000$

Now, for $[NaCl] = 25\times10^{-4}$ M

$$\lambda_m = 127 - 10^3(25\times10^{-4})$$

$$\lambda_m = 127 - 10^3\times5\times10^{-3}$$

$$\lambda_m = 77$$

But $\lambda_m = \dfrac{K\times1000}{M}$ $\qquad K = \left(\dfrac{\ell}{a}\right)\times\dfrac{1}{R}$

$$\lambda_m = \left(\frac{\ell}{a}\right)\times\frac{1}{R}\times\frac{1000}{M}$$

$\lambda_m =$ [cell constant] $\times\dfrac{1000}{R\times M}$ $\qquad \Rightarrow 77 =$ [Cell constant] $\times\dfrac{1000}{1000\times25\times10^{-4}}$

Cell constant $= 77\times25\times10^{-4} = 0.1925\,cm^{-1} = 0.19$ (rounding off)

8.(192.5) For Na_2SO_4 solution

$$K = \left(\frac{\ell}{a}\right) \times \frac{1}{R} = \frac{0.1925}{400} \text{ohm}^{-1} \text{cm}^{-1}; \qquad \lambda_m = \frac{K \times 1000}{M} = \frac{0.1925 \times 1000}{400 \times \frac{5}{2} \times 10^{-3}} = 192.5$$

9.(7) & 10.(6.35)

$$I_2 + 2S_2O_3^{2-} \longrightarrow 2I^- + S_4O_6^{2-}$$

$I_2 = 0.005$ mmoles titrated with excess hypo solution

$(S_2O_3^{2-})_{excess} = 0.01 \text{mmoles}$ (considering the stoichiometry)

$(S_2O_3^{2-})_{total} = 0.11 \text{mmoles}$

$(S_2O_3^{2-})_{reacted}$ with $I_2 = 0.11 - 0.01 = 0.1 \text{mmole}$

$(I_2)_{reacted}$ with $S_2O_3^{2-} = 0.05 \text{mmole}$ (following above stoichiometry)

$(CO)_{reacted} = 0.25 \text{mmole}$; $\quad g_{CO} = 28 \times 0.25 = 7 \text{mg}$

For 20 L $g_{I_2} = 0.05 \times 254$; $\quad$ For 10 L $g_{I_2} = \frac{0.05 \times 254}{2} = 6.35 \text{mg}$

11.(50) $Mn_3O_4 \xrightarrow{\text{dissolved}} Mn^{2+} \xrightarrow{MnO_4^-/Cl^-} Mn^{3+}$

Normality of $KMnO_4$ is 0.15 against oxalate where n-factor of $KMnO_4$ is 5. But in the above reaction n-factor of $KMnO_4$ is 4.

$\therefore$ Normality of $KMnO_4$ in the above reaction is $\frac{4}{5} \times 0.15$

$\therefore$ Equivalent of $KMnO_4 = \frac{4}{5} \times 0.15 \times 25 \times 10^{-3} = 3 \times 10^{-3}$

$\therefore$ Equivalent of Mn^{2+} reacted $= 3 \times 10^{-3}$ $\therefore$ Moles of Mn^{2+} reacted or produced $= 3 \times 10^{-3}$

Moles of Mn_3O_4 in the sample $= \frac{3 \times 10^{-3}}{3} = 10^{-3}$

$\therefore$ % of Mn_3O_4 in the sample $= \frac{10^{-3} \times 229 \times 100}{0.458} = 50\%$

12.(0.44) $HCN + I_3^-(aq) \longrightarrow ICN(aq) + 2I^-(aq) + H^+(aq)$

$$As_4O_6 + 6H_2O \longrightarrow 4H_3AsO_3$$

$$H_3\overset{(+3)}{As}O_3(aq) + I_3^-(aq) \longrightarrow \overset{(+5)}{As}O_4^{3-} + 3I^-$$

1.32 gm of $As_4O_6 = \frac{1.32}{396} \times 4 = \frac{4}{3} \times 10^{-2} =$ moles of I_3^-

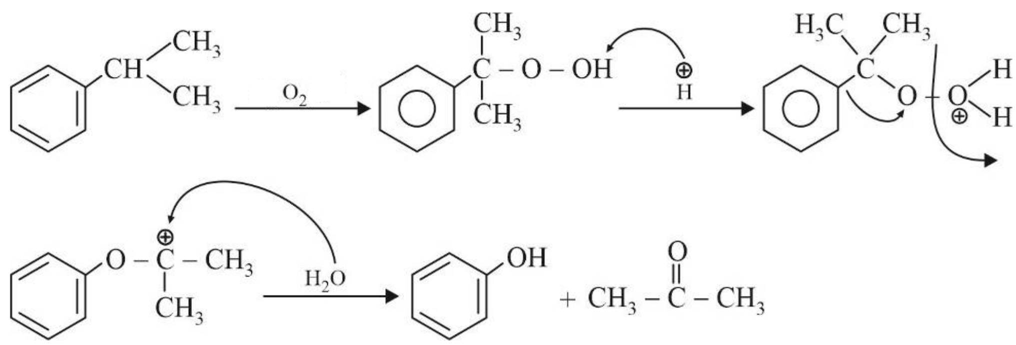
$\therefore$ Concentration of $I_3^- = \frac{4 \times 10^{-2}}{3 \times 10.42} \times 1000 = \frac{40}{10.42 \times 3}$

$\therefore$ In 5.21 ml of I_3^- solution, moles of $I_3^- = \dfrac{40\times 5.21}{10.42\times 3\times 1000}$ = moles of HCN

$\therefore$ Concentration of HCN in blood $= \dfrac{40\times 5.21}{10.42\times 3\times 1000}\times\dfrac{1000}{15} = 0.44$

COMPREHENSION WITH SINGLE TYPE

13.(D)



14.(D) $C_6H_5-N=C=O \xrightarrow{C_3H_7OH} C_6H_5-NHCOOC_3H_7$

15.(C) & 16.(A)

$$2MnO_4^-(Y)+3Mn^{2+}(Z)+2H_2O \longrightarrow 5MnO_2+4H^+$$

In order to reduce MnO_4^-, Mn^{2+} gets oxidised (comproportionation reaction)

$$2KMnO_4(Y)+KBr+H_2O \longrightarrow KBrO_3+2KOH+2MnO_2$$

INTEGER TYPE

17.(3) $T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1} \Rightarrow 300\times V^{1/3}=T_2(8V)^{1/3} \Rightarrow T_2=150\,K$

$W=nC_v(T_2-T_1)=0.5\times 3R\,(150-300)=0.5\times 3\times 2(-150)=-450\,cal$

18.(40) $m\Delta V=\dfrac{h}{\lambda}$; $\quad \dfrac{7\times\Delta V}{6\times 10^{23}\times 10^{3}}=\dfrac{6.6\times 10^{-34}}{\dfrac{990}{7}\times 10^{-9}}$

$\Delta V=0.4\ m/s$; $\quad \Delta V=40\,cm/s$

19.(6) Ostwald's process

1. $NH_3(g)+O_2(g) \xrightarrow[227°C,\ 9\ bar]{Pt/Rh\ gauge\ catalyst} NO(g)+H_2O(g)$

2. $NO(g)+O_2(g) \rightleftharpoons NO_2(g)$

3. $NO_2(g)+H_2O(\ell) \longrightarrow HNO_3(aq)+NO(g)$

$A: NH_3, \quad \therefore x=1$

$B: NO, \quad \therefore y=1$

$C: NO_2, \quad \therefore z=4$

So, $x+y+z=6$

Mathematics

ONE OR MORE THAN ONE CHOICE

1.(ABD) (A) 11 → 4v, 7c $\Rightarrow\ {}^7C_2 \cdot 6! = 3 \cdot 7 \cdot 6! = 3 \cdot 7!$

(B) Number of ways $= \dfrac{15!}{r!(15-r)!} = {}^{15}C_r$ $\quad \underbrace{WW......W}_{r}\ \underbrace{BBB......B}_{15-r}$

This is maximum if $r = 7$ *or* 8

(C) 12 → 4A, 5A, 3 all different $\Rightarrow$ Total no. of combinations $= 5 \cdot 6 \cdot 8 - 1 = 240 - 1 = 239$

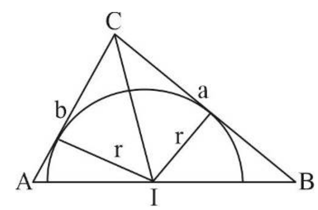
(D) 2 alike + 2 other alike + 2 other alike = 1

2 alike + 2 other alike + 2 different $= {}^3C_2 \cdot {}^4C_2 = 18$

2 alike + 4 different $= {}^3C_1 \cdot {}^5C_4 = 15$

All 6 different = 1; Answer = 35

2.(AC) $\dfrac{1}{2}ra + \dfrac{1}{2}rb = \dfrac{1}{2}ab\sin C$; $\quad r(a+b) = 2\Delta$



$$r = \frac{2\Delta}{a+b} \quad \ldots\text{(i)}$$

$$\therefore\ r = \frac{2abc}{4R(2R\sin A + 2R\sin B)} = \frac{abc}{4R^2(\sin A + \sin B)} \Rightarrow (A)$$

Also, $x = \dfrac{2ab}{a+b}\cos\dfrac{C}{2}$

From (i) $r = \dfrac{2\frac{1}{2}ab\sin C}{a+b} = \dfrac{2ab\sin\frac{C}{2}\cos\frac{C}{2}}{a+b} = \dfrac{2ab\cos\frac{C}{2}}{a+b}\sin\dfrac{C}{2} = x\sin\dfrac{C}{2} \Rightarrow (C)$

3.(AC) Area $(T) = \dfrac{c \cdot c^2}{2} = \dfrac{c^3}{2}$

Area $(R) = \dfrac{c^3}{2} - \displaystyle\int_0^c x^2 dx = \dfrac{c^3}{2} - \dfrac{c^3}{3} = \dfrac{c^3}{6}$ $\qquad \therefore\ \displaystyle\lim_{c \to 0^+} \dfrac{Area(T)}{Area(R)} = \lim_{c \to 0^+} \dfrac{c^3}{2} \cdot \dfrac{6}{c^3} = 3$

4.(ABD) $\dfrac{dy}{dx} + y = f(x)$

I.F. $= e^x$; $\quad$ Now if $0 \le x \le 2$ then $ye^x = \int e^x e^{-x} dx + C \quad \Rightarrow \quad ye^x = x + C$

$x = 0,\ y(0) = 1, \quad C = 1$

$\therefore\ ye^x = x + 1 \qquad \ldots\text{(i)}$

$y = \dfrac{x+1}{e^x}$; $y(1) = \dfrac{2}{e}$

$y' = \dfrac{e^x - (x+1)e^x}{e^{2x}}$; $\quad y'(1) = \dfrac{e - 2e}{e^2} = \dfrac{-e}{e^2} = -\dfrac{1}{e}$

If $x > 2$; $\quad ye^x = \int e^{x-2}dx$

$ye^x = e^{x-2} + C$; $\quad y = e^{-2} + Ce^{-x}$

As y is continuous $\quad \therefore \lim\limits_{x \to 2} \dfrac{x+1}{e^x} = \lim\limits_{x \to 2}\left(e^{-2} + Ce^{-x}\right)$

$3e^{-2} = e^{-2} + Ce^{-2} \Rightarrow C = 2 \quad \therefore$ for $x > 2$

$y = e^{-2} + 2e^{-x}$ hence $y(3) = 2e^{-3} + e^{-2} = e^{-2}(2e^{-1} + 1)$

$y' = -2e^{-x}$; $\quad y'(3) = -2e^{-3}$

5.(BCD) $\overrightarrow{AB} = \bar{b} - \bar{a} = 4\hat{i} - 12\hat{j} - 3\hat{k}$; $\quad \overrightarrow{CD} = \bar{d} - \bar{c} = 3\hat{i} + 4\hat{j} - 12\hat{k}$

$\overrightarrow{AC} = \bar{c} - \bar{a} = 3\hat{i} - 7\hat{j} + 2\hat{k}$; $\quad \overrightarrow{BD} = \bar{d} - \bar{b} = 2\hat{i} + 9\hat{j} - 7\hat{k}$

By definition $d = \dfrac{\left(\overrightarrow{AB} \times \overrightarrow{CD}\right) \cdot \overrightarrow{AC}}{\left|\overrightarrow{AB} \times \overrightarrow{CD}\right|}$ ...(i)

$= \dfrac{\left(\overrightarrow{AB} \times \overrightarrow{CD}\right) \cdot \overrightarrow{BD}}{\left|\overrightarrow{AB} \times \overrightarrow{CD}\right|}$ ...(ii)

$\overrightarrow{AB} \times \overrightarrow{CD} = 13(12\hat{i} + 3\hat{j} + 4\hat{k}) \quad \therefore \left|\overrightarrow{AB} \times \overrightarrow{CD}\right| = 169$

$\therefore \quad d = \dfrac{13(12\hat{i} + 3\hat{j} + 4\hat{k})}{169} \cdot (3\hat{i} - 7\hat{j} + 2\hat{k}) = \dfrac{23}{13}$ using (i)

Also, $\therefore \quad d = \dfrac{13(12\hat{i} + 3\hat{j} + 4\hat{k})}{169} \cdot (2\hat{i} - 9\hat{j} + 7\hat{k}) = \dfrac{23}{13}$ using (ii)

6.(ABD) We have $2h = t_3^2 + 2$ ...(i)

$2k = t_3$ ...(ii)

$\therefore 2h = 4k^2 + 2 \quad \therefore 2y^2 = x - 1$

$y^2 = \dfrac{1}{2}(x-1)$ (Parabola); Now interpret

COMPREHENSION WITH NUMERICAL TYPE

7.(0) & 8.(81)

We have $\dfrac{3x + 6y}{k} = 1$ ...(i)

$2x^2 + 2xy + 3y^2 - 1 = 0$ ...(ii)

Now homogenizing (ii) with the help of (i), we get

$\Rightarrow 2x^2 + 2xy + 3y^2 - \left(\dfrac{3x+6y}{k}\right)^2 = 0 \quad \Rightarrow k^2(2x^2 + 2xy + 3y^2) - (3x + 6y)^2 = 0$

Now coefficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow (2k^2 - 9) + (3k^2 - 36) = 0 \Rightarrow 5k^2 = 45 \Rightarrow k^2 = 9 \Rightarrow k = 3 \; or \; -3$

9.(5049) If $b=1$

$$f(x)=8x^3+4ax^2+2x+a$$

$$f'(x)=24x^2+8ax+2 \quad \text{or} \quad 2(12x^2+4ax+1)$$

For non-monotonic $f'(x)=0$ must have distinct roots

Hence $D>0$ i.e., $16a^2-48>0 \Rightarrow a^2>3$; $\therefore$ $a>\sqrt{3}$ *or* $a<-\sqrt{3}$

$\therefore$ $a\in 2,3,4,.........$; Sum = 5050 – 1 = 5049

10.(256) If x_1, x_2 & x_3 are the roots then $\log_2 x_1+\log_2 x_2+\log_2 x_3=5$

$$\log_2(x_1x_2x_3)=5; \qquad x_1x_2x_3=32; \qquad -\frac{a}{8}=32 \Rightarrow -a=256$$

11.(4) $Y-y=m(X-x)$

Put $Y=0$; $\quad X=x-\dfrac{y}{m}$

Hence the quantity

$x-\dfrac{y}{m}$ is same for both the curves

$$x_1-\frac{y_1}{m_1}=x_2-\frac{y_2}{m_2} \quad (\text{but } x_1=x_2)$$

$$\frac{y_1}{m_1}=\frac{y_2}{m_2} \text{ or } \frac{y_1}{y_2}=\frac{m_1}{m_2}$$

$$\frac{f(x)}{\int_{-\infty}^{x} f(t)\,dt}=\frac{f'(x)}{f(x)} \qquad \ldots\text{(i)}$$

Integrating equation (i), we get; $\ln\left(\int_{-\infty}^{x} f(t)\,dt\right)=\ln(f(x))+C$

If $x=0$, $\int_{-\infty}^{0} f(t)\,dt=\dfrac{1}{2}$ & $f(0)=1 \Rightarrow C=\ln\left(\dfrac{1}{2}\right)=-\ln 2$

$$\therefore \ln\left(\int_{-\infty}^{x} f(t)\,dt\right)=\ln\left(\frac{f(x)}{2}\right) \qquad \therefore f(x)=2\int_{-\infty}^{x} f(t)\,dt \Rightarrow \frac{f(x)}{\int_{-\infty}^{x} f(t)\,dt}=2 \quad \ldots\text{(ii)}$$

Integrating equation (ii), we get $\ln\left(\int_{-\infty}^{x} f(t)\,dt\right)=2x+C$

$$x=0 \;\; -\ln 2=C; \qquad \ln\left(2\left(\int_{-\infty}^{x} f(t)\,dt\right)\right)=2x$$

$$2\int_{-\infty}^{x} f(t)\,dt = e^{2x}$$

$$2f(x) = 2e^{2x} \qquad \ldots\text{(iii)}$$

Differentiating both sides of equation (iii), we get

$$f(x) = e^{2x}; \qquad \lim_{x\to 0}\frac{f^2(x)-1}{x} = \lim_{x\to 0}\frac{e^{4x}-1}{x} = 4$$

12.(9) $Y = y - m(X - x)$

Put $Y = 0$; $\quad X = x - \dfrac{y}{m}$

Hence the quantity $\quad x - \dfrac{y}{m}$ is same for both the curves

$$x_1 - \frac{y_1}{m_1} = x_2 - \frac{y_2}{m_2} \text{ (but } x_1 = x_2\text{)}; \qquad \frac{y_1}{m_1} = \frac{y_2}{m_2} \text{ or } \frac{y_1}{y_2} = \frac{m_1}{m_2}$$

$$\frac{f(x)}{\int_{-\infty}^{x} f(t)\,dt} = \frac{f'(x)}{f(x)} \qquad \ldots\text{(i)}$$

Integrating equation (i), we get ; $\ln\left(\int_{-\infty}^{x} f(t)\,dt\right) = \ln(f(x)) + C$

If $x = 0$ $\quad \ln\int_{-\infty}^{0} f(t)\,dt = \dfrac{1}{2}$ & $f(0) = 1 \Rightarrow C = \ln\left(\dfrac{1}{2}\right) = -\ln 2$

$$\therefore \ \ln\left(\int_{-\infty}^{x} f(t)\,dt\right) = \ln\left(\frac{f(x)}{2}\right) \qquad \therefore \ f(x) = 2\int_{-\infty}^{x} f(t)\,dt \Rightarrow \frac{f(x)}{\int_{-\infty}^{x} f(t)\,dt} \qquad \ldots\text{(ii)}$$

Integrating equation (ii), we get $\ln\left(\int_{-\infty}^{x} f(t)\,dt\right) = 2x + C$

$$x = 0 \ -\ln 2 = C; \qquad \ln\left(2\left(\int_{-\infty}^{x} f(t)\,dt\right)\right) = 2x; \qquad 2\int_{-\infty}^{x} f(t)\,dt = e^{2x}$$

$$2f(x) = 2e^{2x} \qquad \ldots\text{(iii)}$$

Differentiating both sides of equation (iii), we get

$$f(x) = e^{2x}; \qquad f'(0) = 2$$

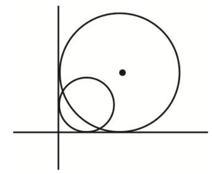
Tangent at (0,1) is $y - 1 = 2x$

These lines meet the x-axis at $\left(-\dfrac{1}{2}, 0\right)$ & (2,0) $\therefore$ $\quad$ Area $= \dfrac{1}{2}\left(2 + \dfrac{1}{2}\right)1 = \dfrac{5}{4}$

COMPREHENSION WITH SINGLE TYPE

13.(A) $\text{LHD} = f'(0^-) = \lim_{h\to 0} \dfrac{\dfrac{-\sin h + \tan h + \cos h - 1}{2h^2 + \ln(2-h) - \tan h} - 0}{-h} = \lim_{h\to 0} \dfrac{\dfrac{\sin h}{h} - \dfrac{\tan h}{h} + \dfrac{1-\cos h}{h^2} \times h}{-2h^2 + \ln(2-h) - \tan h} = 0$

$RHD = f'(0^+) = \lim_{h\to 0} \dfrac{e^{h^2} - 1 - 0}{h} = h \times \dfrac{e^{h^2} - 1}{h^2} = 0$



$L_1 \equiv y = 0\ \&\ L_2 \equiv x = 0$

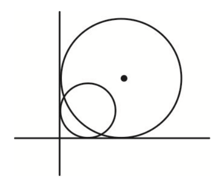
$(x-r)^2 + (y-r)^2 = r^2$ (family of circle)

$x^2 + y^2 - 2rx - 2ry + r^2 = 0$; $\quad 2(r_1r_2 + r_1r_2) = r_1^2 + r_2^2$ *or* $4r_1r_2 = r_1^2 + r_2^2$

$\left(\dfrac{r_2}{r_1}\right)^2 - 4\left(\dfrac{r_2}{r_1}\right) + 1 = 0$; $\quad \dfrac{r_2}{r_1} = \dfrac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$

14.(C) $\text{LHD} = f'(0^-) = \lim_{h\to 0} \dfrac{\dfrac{-\sin h + \tan h + \cos h - 1}{2h^2 + \ln(2-h) - \tan h} - 0}{-h} = \lim_{h\to 0} \dfrac{\dfrac{\sin h}{h} - \dfrac{\tan h}{h} + \dfrac{1-\cos h}{h^2} \times h}{-2h^2 + \ln(2-h) - \tan h} = 0$

$RHD = f'(0^+) = \lim_{h\to 0} \dfrac{e^{h^2} - 1 - 0}{h} = h \times \dfrac{e^{h^2} - 1}{h^2} = 0$

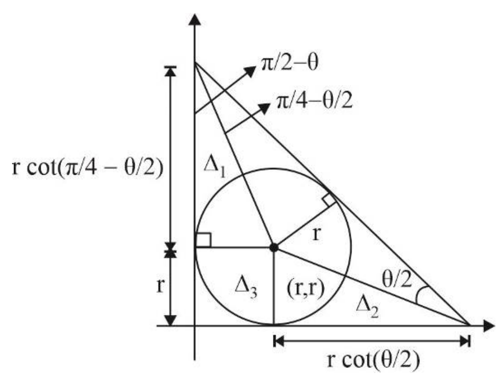


$L_1 \equiv y = 0\ \&\ L_2 \equiv x = 0$

$\text{LHD} = f'(0^-) \lim_{h\to 0} \dfrac{\dfrac{-\sin h + \tan h + \cos h - 1}{2h^2 + \ln(2-h) - \tan h} - 0}{-h} = \lim_{h\to 0} \dfrac{\dfrac{\sin h}{h} - \dfrac{\tan h}{h} + \dfrac{1-\cos h}{h^2} \times h}{-2h^2 + \ln(2-h) - \tan h} = 0$

$f'(0^+) = RHD == f'(0^-) \lim_{h\to 0} \dfrac{e^{h^2} - 1 - 0}{h} = h \times \dfrac{e^{h^2} - 1}{h^2} = 0$

$L_1 \equiv y = 0\ \&\ L_2 \equiv x = 0$; $\quad \Delta = 2[\Delta_1 + \Delta_2 + \Delta_3]$



$\Delta = 2 \times \dfrac{1}{2}\left(\cot\left(\dfrac{\pi}{4} - \dfrac{\theta}{2}\right) + \cot\dfrac{\theta}{2} + 1\right)$ $\quad$ (using $\dfrac{1}{2}ab$]

$\Delta = \dfrac{\cos\left(\dfrac{\pi}{4} - \dfrac{\theta}{2}\right)}{\sin\left(\dfrac{\pi}{4} - \dfrac{\theta}{2}\right)} + \dfrac{\cos\dfrac{\theta}{2}}{\sin\dfrac{\theta}{2}} + 1$

$\Delta = 1 + \dfrac{2\sin\dfrac{\pi}{4}}{2\sin\dfrac{\theta}{2} \cdot \sin\left(\dfrac{\pi}{4} - \dfrac{\theta}{2}\right)}$ $\qquad \Delta = 1 + \dfrac{\sqrt{2}}{\cos\left(\theta - \dfrac{\pi}{4}\right) - \cos\left(\dfrac{\pi}{4}\right)}$

Δ is minimum if numerator is maximum when $\theta = \dfrac{\pi}{4}$

$\Delta_{\min} = 1 + \dfrac{\sqrt{2}}{1 - \dfrac{1}{\sqrt{2}}} = 1 + \dfrac{2}{\sqrt{2} - 1} = 1 + 2\left(\sqrt{2} + 1\right) = 3 + 2\sqrt{2}$

15.(C) & 16.(D)

$$ax^2 + 2bx + b = 5x^2 - 3bx - a \Rightarrow (a-5)x^2 + 5bx + (b+a) = 0$$

If $a \neq 5$ then since $x \in R$

$$D = 25b^2 - 4(b+a)(a-5) \geq 0 \ \forall \ b \in R \qquad \Rightarrow 25b^2 - 4(a-5)b - 4a(a-5) \geq 0 \ \forall b \in R$$

$$\therefore \ 16(a-5)^2 + 16(25)a(a-5) \leq 0$$

$$\Rightarrow 16(a-5)(a-5+25a) \leq 0 \Rightarrow (a-5)(26a-5) \leq 0 \quad \therefore \ a \in \left[\frac{5}{26}, 5\right)$$

If $a = 5$, $\qquad 5bx + (b+5) = 0$

Not satisfied for $b = 0 \ \therefore \ a_m \in \{1,2,3,4\}$

$$t_r = (r-1)(r-2)(r-3)(r-4); \qquad S_n = \frac{1}{5}\sum_{r=1}^{n}(r-4)(r-3)(r-2)(r-1)[r-(r-5)]$$

$$= \frac{1}{5}\sum_{r=1}^{n}((r-4)(r-3)(r-2)(r-1)r - (r-5)(r-4)(r-3)(r-2)(r-1))$$

$$= \frac{1}{5}n(n-1)(n-2)(n-3)(n-4)$$

$S_n = 0 \Rightarrow n = 1,2,3,4$ $\qquad$ n = 0 (rejected)

$$\Sigma\frac{1}{t_r} = \frac{1}{3}\lim_{n\to\infty}\sum_{r=5}^{n}\frac{(r-1)-(r-4)}{(r-4)(r-3)(r-2)(r-1)}$$

$$= \frac{1}{3}\lim_{n\to\infty}\sum_{r=5}^{n}\left(\frac{1}{(r-4)(r-3)(r-2)} - \frac{1}{(r-3)(r-2)(r-1)}\right)$$

$$= \lim_{n\to\infty}\frac{1}{3}\left[\frac{1}{6} - \frac{1}{(n-3)(n-2)(n-1)}\right] = \frac{1}{18}$$

INTEGER TYPE

17.(17) Number of ways in which A,B,C can be given 3 tiles

3 equal groups of 9 tiles and distributed $= n(E) = \dfrac{9!\,3!}{(3!)^3\,3!} = 1680$

Now we have 1,3,5,7,9 (odd numbers) : 2,4,6,8 (even numbers)

In order that all the 3 persons must have an odd total. One person must have all the 3 tiles marked with odd number and each of the remaining two persons must have 2 tiles with even marked and 1 tiles with odd marked

$$\therefore \ n(E) = \underbrace{{}^3C_1}_{\text{select1 person}} \cdot \underbrace{{}^5C_3}_{\text{3 odd tile}} \cdot \underbrace{{}^2C_1}_{\text{1 od d to each}} \cdot \underbrace{\frac{4!\cdot 2!}{2!\cdot 2!\cdot 2!}}_{\text{grouping 4 even tiles in two equal groups \& distribute}} = 3\cdot 10\cdot 2\cdot 6 = 360$$

$$P(E) = \frac{360}{1680} = \frac{9}{42} = \frac{3}{14} = \frac{m}{n}; \qquad \text{Hence } m+n = 3+14 = 17$$

18.(2) Any point on the parabola $y^2 = 4ax$ is $(at^2, 2at)$. Equation of chord of the ellipse $\frac{x^2}{2a^2} + \frac{y^2}{a^2} = 1$, whose mid-point is $(at^2, 2at)$ is $\frac{x \cdot at^2}{2a^2} + \frac{y \cdot 2at}{a^2} = \frac{a^2t^4}{2a^2} + \frac{4a^2t^2}{a^2}$

$\Rightarrow \quad tx + 4y = at^3 + 8at \quad (\because \ t \neq 0)$

As it passes through $\left(11a, -\frac{a^2}{4}\right) \quad \Rightarrow \quad 11at - 4\left(\frac{a^2}{4}\right) = at^3 + 8at \Rightarrow at^3 - 3at + a^2 = 0$

$\Rightarrow \quad t^3 - 3t + a = 0 \quad (a \neq 0)$

Now, three chords of the ellipse will be bisected by the parabola if the equation (i) has three real and distinct roots.

Let $\quad f(t) = t^3 - 3t + a$

$\quad f'(t) = 3t^2 - 3 = 0 \Rightarrow t = \pm 1$

So, $\quad f(1)f(-1) < 0 \quad \Rightarrow \quad a \in (-2, 2)$

But $\quad a \neq 0$, so $a \in (-2,0) \cup (0,2) \quad \therefore \quad$ Number of integral values of ‘a’ = 2

19.(11) $T_r = \frac{1}{\sqrt{\frac{r}{n}} \cdot n\left(3\sqrt{\frac{r}{n}} + 4\right)^2}$, $\quad S = \frac{1}{n}\sum_{r=1}^{4n} \frac{1}{\left(3\sqrt{\frac{r}{n}} + 4\right)^2 \cdot \sqrt{\frac{r}{n}}} = \int_0^4 \frac{dx}{\sqrt{x}\left(3\sqrt{x} + 4\right)^2}$

put $3\sqrt{x} + 4 = t \Rightarrow \frac{3}{2}\frac{1}{\sqrt{x}}dx = dt$

$= \frac{2}{3}\int_4^{10} \frac{dt}{t^2} = \frac{2}{3}\left[\frac{1}{t}\right]_{10}^{4} = \frac{2}{3} \cdot \frac{6}{40} = \frac{1}{10} \Rightarrow m = 1, \ n = 10 \Rightarrow m + n = 11$